First order predicate logic

When same assertive sentence will associate with different 'Subjects', then we have problem to ~~define~~ write that sentence in propositional form.

For example, peter is a man, paul is a man

~~So~~ To convert this ~~sence~~ sentences into propositional form we can take a propositional variable with some argument, - then it is called a predicate,

For example x is man = $M(x)$.

where $M(x)$ → M is a proposition with argument x

therefore $M(x)$ is called a predicate.

therefore predicate logic is a logical extension of propositional logic, where we can also quantify the sentences.

Predicate calculus.

It is an extension of language of propositional Calculus and has three more notions.

- Terms
- Predicates
- Quantifiers like for all and there exists.

symbols and Conventions used in Predicate Calculus:

1) Predicate symbols → P, Q, R etc.

2) Function symbols → f(), g(), h() ~~etc~~, mary, father etc.

3) Variable symbols → x, y, z, etc.

4) Constants → like names of objects, john, mary etc.

5) Quantifiers: $\forall$ for all, $\exists$ there exists.

6) logical operators $\sim, \wedge, \vee, \rightarrow, \leftrightarrow$.

If function or predicate takes n arguments then it is called n-place function or n-place predicate.

(12) logical notions in Predicate Calculus:-

~~To~~ Three logical notions are extra in predicate calculus. they are, term, predicate and quantifiers.

Defⁿ:- A ~~con~~ term is defined recursively as follows:

- A constant is a term
- A variable is a term that stands for different ~~individual~~ Individuals.
- If f is n-place function symbols and $t_1, t_2, \ldots t_n$ are terms then $f(t_1, \ldots t_n)$ is a term.
- All terms are generated by applying above rules.

Defⁿ:- A term is called ground term if it is free from variables. For eg:

- Constant is ground term
- Functions with ground terms as arguments is a ground term.

Defⁿ: A function is a mapping that maps n terms to a term. Mathematically it is defined as $f(t_1, t_2, \ldots t_n) \in \Gamma$, $\Gamma^n \to \Gamma$ such that where Γ is a set of terms defined as previous.

Defⁿ:- A predicate is a relation that maps n terms to a truth value true (T) or false (F). ~~There fore~~ mathematically it is written as.

$$P: \Gamma^n \to \{T, F\}.$$

Eg:- A statement x is greater than y can be represented as,

Greater(x, y) = T if $x > y$
= F if $x < y$ or otherwise.

Eg:- A statement john's father loves john is represented as Love(father(john), john). Here father is a function that's maps john to his father.

Quantifiers

The variables which are used in conjunction with quantifiers are of two types.

1) There exists, denoted as $\exists$
2) for all denoted as $\forall$

Eg: john loves everyone is represented as.
Lets define the predicate as follows:

Love (x, y) : x has love for y.

then the sentence is represented as.

$\forall y$ Love (john, y).

First order predicate calculus (FOPC):-

It is one in which the quantification is used on simple variables not on predicates or functions.

If the quantification is over the first order predicates and functions then it becomes Second order predicates, Like this way third order predicates ~~are in our syllabus but~~ we can define. Higher order predicates can also exists.

The first order predicate calculus is a formal language. It is defined by its syntax — alphabets, symbols and rules which form a legitimate expression.

The ~~leg~~ legitimate expression of FOPC are called well formed formula.

Defn:- Well formed formula in FOPC is defined recursively as follows:

- Atomic formula $P(t_1, t_2 -- t_n)$ is a well formed formula. atomic formula is also called atom in short. Here P is predicate symbol t_i is the ith term.
- If α and β are well formed formulae then $\sim\alpha, \alpha \vee \beta, \alpha \wedge \beta, \alpha \rightarrow \beta$ and $\alpha \leftrightarrow \beta$ are well formed formulae.

- If α is a well formed formula and x is free variable in α, then $(\forall x)\alpha$ and $(\exists x)\alpha$ are well formed formulae
- Well-formed formulae are generated by a finite number of application of above rules.

Eg: Formulate FOPC formulae of the following sentences

1. Every natural number is an integer number
2. There exists a number that is even number
3. For every number x, there exists a number y such that $x < y$
4. Every woman loves a child.

Solution Let us define the following predicates

1) Natural (x) — x is a natural number
2) Integer (x) — x is an integer
3) Even (x) — x is an even number
4) Woman (x) — x is a woman
5) child (x) — x is a child
6) love (x, y) — x loves y
7) less (x, y) — x is less than y

Therefore above sentences can be written as follows.

1. $(\forall x)(\text{Natural}(x) \rightarrow \text{Integer}(x))$
2. $(\exists x)\ \text{Even}(x)$.
3. $(\forall x)(\exists y)\ \text{less}(x, y)$
4. $(\forall x)(\text{Woman}(x) \rightarrow \text{loves}(x, y) \wedge \text{child}(y)))$

Eg: Translate the text "Every man is mortal. John is a man. Therefore John is mortal into FOPC formula.

Solution. Let us define the following

~~Ex~~ Man (x) — x is a man
Mortal (x) — x is mortal.

Then
"Every man is mortal : $(\forall x)\,(\text{Man}(x) \rightarrow \text{Mortal}(x))$

John is a man : Man (john).

John is mortal : mortal (john).

Therefor while text can be written as.

$$(\forall x)\big((\text{Man}(x) \rightarrow \text{Mortal}(x)) \wedge \text{Man}(\text{john})\big) \rightarrow \text{mortal}(\text{john})$$

Precedance of operators.

The logical operators and quantifiers are listed according to higher to lower priority are as below:

$\sim, \forall, \exists, \wedge, \vee, \rightarrow, \leftrightarrow$.

The quantifiers $\forall, \exists$ has same priority as $\sim$. And the parantheris has the highest priority.

The range over which a variable is valid is called its scope.

Defn:- ~~An~~ An occurance of a variable x in a formula is said to bound if and only if the occurance is within the scope of a quantifier employing the variable x, otherwise is said to be free.

Eg: $\forall x\, P(x, y)$, where x is bound but y is free in $P(x, y)$.

Eg. $(\exists x)\,(\forall y)\;\; P(x, y) \wedge Q(x)$

where x, y both are bound in $P(x, y)$

but x is free in $Q(x)$ because $Q(x)$ is outside of scope of $(\exists x)\,(\forall y)$.