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Q. Eg prove that $\vdash p \rightarrow \sim\sim p$ is a theorem,

Q. Eg. $\{p \rightarrow q, \sim q\} \vdash \sim p$ prove that called Modus Tollens.

Ans prove $\{p \rightarrow q, \sim q\}$

hypothesis $p \rightarrow q$ — (1)

hypothesis $\sim q$ — (2)

theorem $\sim\sim p \rightarrow p$ — (3)

$\therefore$ By transitivity $\sim\sim p \rightarrow q$ — (4) [By 3, 1]

theorem $q \rightarrow \sim\sim q$ — (5)

$\therefore$ By transitivity 4,5, $\sim\sim p \rightarrow \sim\sim q$ — (6)

By axiom 3, 6. $(\sim\sim p \rightarrow \sim\sim q) \rightarrow (\sim q \rightarrow \sim p)$ — (7)

From 6, 7, Modus Ponen $\sim q \rightarrow \sim p$ — (8)

By 2, 8, Modus Ponen $\sim p$ — (9)

Hence $\{p \rightarrow q, \sim q\} \vdash \sim p$ proved

Q. Eg $\{p\} \vdash (\sim q \rightarrow \sim\sim p)$

A prove $\{p\} \vdash (\sim q \rightarrow \sim\sim p)$

hypothesis p — ①

By Axiom 1 $p \rightarrow (\sim q \rightarrow p)$ — ②

From ①② , Modus Ponens $\sim q \rightarrow p$ — ③

Theorem $p \rightarrow \sim\sim p$ — ④

By 3, 4, Transitivity $\sim q \rightarrow \sim\sim p$ — ⑤

Hence $\{p\} \vdash (\sim q \rightarrow \sim\sim p)$ proved.

Q. Show that $\{p\} \vdash (\sim p \rightarrow q)$.

A. prove $\{p\} \vdash (\sim p \rightarrow q)$

hypothesis p ①

from axiom 1 $\sim\sim p \rightarrow (\sim q \rightarrow \sim\sim p)$ — ②

theorem $p \rightarrow \sim\sim p$ — ③

By 2, 3, Transitivity $p \rightarrow (\sim q \rightarrow \sim\sim p)$ — ④

From ①, 4, modus ponens $\sim q \rightarrow \sim\sim p$ — ⑤

By axiom 3 $\sim p \rightarrow q$ — ⑥

Deduction theorem

If Σ is a set of hypotheses and α and β are well formed formulae then $\{\Sigma \cup \alpha\} \vdash \beta$ implies $\Sigma \vdash (\alpha \rightarrow \beta)$

Proof Given $\{\Sigma, \cup \alpha\} \vdash \beta$ where $\Sigma = \{\lambda_1, \lambda_2, \ldots \lambda_n\}$ a set of ~~wffs~~ hypotheses. α, β are wff. We have to prove $\Sigma \vdash (\alpha \rightarrow \beta)$..

There are two cases

Case 1 :→ $\beta \in \Sigma$ or β is a ~~ax~~ axiom.

Case 2 → β is derived from λ_i and $\lambda_i \rightarrow \beta$

for case 1:

Axiom / hypotheses β --- ①

By Axiom 1 $\beta \to (\alpha \to \beta)$ --- ②

By Modus ponen 1, 2 $\alpha \to \beta$

Hence $\Sigma \vdash (\alpha \to \beta)$ is proved.

for case 2:

hypothesis λ_i --- ①

Deduction assumed $\lambda_i \to \beta$ --- ②

Axiom 1. $\lambda_i \to (\alpha \to \lambda_i)$ --- ③

By MP ①, ③ $\alpha \to \lambda_i$ --- ④

By Axiom A1. $(\lambda_i \to \beta) \to (\alpha \to (\lambda_i \to \beta))$ --- ⑤

from 2, 5, Modus ponen $\alpha \to (\lambda_i \to \beta)$ --- ⑥

By Axiom 2. $(\alpha \to (\lambda_i \to \beta)) \to ((\alpha \to \lambda_i) \to (\alpha \to \beta))$ --- ⑦

from 6, 7, modus ponen $(\alpha \to \lambda_i) \to (\alpha \to \beta)$ --- ⑧

By 4, 8, Modus ponen $(\alpha \to \beta)$ --- ⑨

Hence $\Sigma \vdash (\alpha \to \beta)$ proved.

Converse of deduction theorem

Given $\Sigma \vdash (\alpha \to \beta)$ prove that $\{\Sigma \cup \alpha\} \vdash \beta$

proof Deduced from Σ $\alpha \to \beta$. given ①

hypothesis assumed α --- ②

$\therefore$ By ①, ② & M.P we have β.

$\therefore \{\Sigma \cup \alpha\} \vdash \beta$ proved

Eg. Prove by deduction theorem

$\{(\sim\sim P \to \sim\sim Q)\} \vdash (P \to Q)$

Ans. Prove $\{\sim\sim P \to \sim\sim Q\} \vdash (P \to Q)$

hypotheses $(\sim\sim P \to \sim\sim Q)$ . . . (1)

By axiom 3 $(\sim\sim P \to \sim\sim Q) \to (\sim Q \to \sim P)$ . . . (2)

By (1), (2), Modus ponen (3)

$\sim Q \to \sim P$

By axiom 3 $(\sim Q \to \sim P) \to (P \to Q)$ (4)

By 3, 4, Modus ponen. $(P \to Q)$. (5)

Hence $\{(\sim\sim P \to \sim\sim Q)\} \vdash (P \to Q)$ proved.

Eg 2 $\sim P \to (P \to Q)$ proof that by deduction theorem.

prove $(\sim P) \vdash (P \to Q)$

Hypothesis $\sim P$ . . . (1)

By Axiom 1 $\sim P \to (\sim Q \to \sim P)$ . . . (2)

By (1), (2), Modus ponence $(\sim Q \to \sim P)$ . . . (3)

By Axiom 3. $(\sim Q \to \sim P) \to (P \to Q)$ . . (4)

By 3, 4, Modus ponence $(P \to Q)$ . . (5)

Hence $\sim P \vdash (P \to Q)$ proved.

Eg3. establish $\{P, Q \to (P \to R)\} \vdash (Q \to R)$ using deduction theorem

Prove $\{P, Q \to (P \to R)\} \vdash (Q \to R)$ equivalently $P, Q \to (P \to R), Q \vdash R$

Hypothesis P . . . (1)

" $Q \to (P \to R)$ . . . (2)

let " Q . . . (3)

By, 2, 3, M.P) $P \to R$ . . . (4)

By (1, 4, M.P) R . . . 5

Hence proved $Q \to R$

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By Axiom A $R \to (Q \to R)$ ... (6)

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From 5, 6 M.P $(Q \to R)$ ... (7)

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NAME therefore $\{P, (Q \to (P \to R))\} \vdash (Q \to R)$. proved.

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Def^n

A truth valuation is a ~~function~~ mapping from a set of wff's to the set $\{T, F\}$, such that for any wff α, β.

$\therefore V(\sim\alpha) \neq V(\alpha)$

$V(\alpha \to \beta) = F$ iff $V(\alpha) = T$ & $V(\beta) = F$.

~~Def : Soundness and Completeness in Axiomatic System~~

~~Theorem: If α is a formula in Axiomatic System then α is a theorem iff α is valid.~~

~~Proof~~

Semantic tableaux

It is a system of building proofs of formula systematically by using set of semantic rules that are applied on formulae to establish that the formulae are ~~either~~ consistent or inconsistent.

Semantic tableau

It is a binary tree constructed by using semantic tableaux rules with a formula as root.

Let α, β be any two formulae and P, Q, R be atoms.