(5)

DURGAPUR INSTITUTE OF ADVANCED TECHNOLOGY & MANAGEMENT

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They are

Axiom 1 :-> $\alpha \to (\beta \to \alpha)$

Axiom 2 :-> $(\alpha \to (\beta \to \gamma)) \to ((\alpha \to \beta) \to (\alpha \to \gamma))$

Axiom 3 :-> $(\sim\alpha \to \sim\beta) \to (\beta \to \alpha)$

6. The rule of inference for axiomatic System called Modus ponen - is defined as follows:

β is a direct consequence of $\alpha \to \beta$ and α

This can be re-written as.

for: '$\alpha \to \beta$,' and 'α'

logical Consequence : 'β'

Deduction in axiomatic System

A deduction of a formula in Axiomatic System is a sequence of wff's $\alpha_1, \alpha_2, \ldots, \alpha_n$ such that for each i $(1 \le i \le n)$ either α_i is an axiom or hypothesis or is derived from α_j, α_k where $j, k < i$ using modus ponen. inference rule.

Deductive consequence

If Σ is a set of hypothesis (formula assumed to be proved) involved in the deduction of α, then α is called a deductive consequence of Σ or α is deducible from Σ. Written as $\Sigma \vdash \alpha$.

Theorem

If a formula α is deduced from axioms only and no hypothesis are used then α is called a theorem. Written as $\vdash \alpha$, we assume that here 'Σ' is empty.

Eg: $\{Q\} \vdash (P \to Q)$ i.e '$P \to Q$' is logical consequence of $\{Q\}$, → proof it.

Ans. Hypothesis given, Q - - - ①

Axiom 1 $Q \to (P \to Q)$ - - ②

$(P \to Q)$ - - - ③

From 1, 2, Modus ponen

Hence we can conclude that $\{Q\} \vdash (P \to Q)$.

Eg: $\{P \to Q, Q \to R\} \vdash (P \to R)$ proof that.

called chain rule

Ans. Hypothesis given $(P \to Q)$ - - ①

Hypothesis given $(Q \to R)$ - - ②

From axiom 1. $(Q \to R) \to (P \to (Q \to R))$ - ③

[Let α is $Q \to R$ and β is P]

From Modus ponen & 2, 3 $P \to (Q \to R)$ - ④

From axiom 2 $(P \to (Q \to R)) \to ((P \to Q) \to (P \to R))$ - - ⑤

From 4, 5, Modus Ponen $(P\to Q)\to(P\to R)$ ---- (6)

From 1, 6, modus ponen $(P\to R)$ --- (7)

Hence we can conclude $\{P\to Q,\ Q\to R\}\vdash(P\to R)$

Eg: Proof that $\{P\to Q\}\vdash((R\to P)\to(R\to Q))$

Proof - hypothesis given $(P\to Q)$ --- (1)

from Axiom 1. $(P\to Q)\to(R\to(P\to Q))$ -- (2)

From 1, 2, modus ponen $R\to(P\to Q)$ --- (3)

from Axiom 2: $(R\to(P\to Q))\to((R\to P)\to(R\to Q))$ -- (4)

From 3, 4, modus ponen $(R\to P)\to(R\to Q)$ -- (5)

Hence we can conclude that

$$\{P\to Q\}\vdash((R\to P)\to(R\to Q))$$

Eg: Prove that $\vdash(P\to P)$

$\vdash(P\to P)$ --- (a)

Ans. theorem

From Axiom 1 $P\to((Q\to P)\to P)$ -- (1)

From Axiom 2 $(P\to((Q\to P)\to P))\to((P\to(Q\to P))\to(P\to P))$ -- (2)

From (1) (2), modus ponen $(P\to(Q\to P))\to(P\to P)$ -- (3)

From (a), Axiom 1 $P\to(Q\to P)$ --- (4)

From 3, 4, modus ponen $(P\to P)$.

Therefore we can conclude that $\vdash(P\to P)$

So, $\vdash(P\to P)$ is a theorem.

Eg:- Prove that $\vdash (\sim\sim p \to \sim\sim p)$.

It's same as previous example.

Eg: Prove that $\vdash (\sim\sim p \to p)$ is a theorem

Ans. Theorem $\vdash (\sim\sim p \to p)$

By Axiom 1. $\sim\sim p \to (\sim\sim\sim\sim p \to \sim\sim p)$ --- (1)

By Axiom 3. $(\sim(\sim\sim\sim p) \to \sim(\sim p)) \to (\sim p \to \sim\sim\sim p)$ --- (2)

By Transitivity 1, 2, $\sim\sim p \to (\sim p \to \sim\sim\sim p)$ --- (3)

~~By Axiom 3~~ ~~By Axiom~~ $\cancel{\sim\sim p \to ((\sim p \to \sim(\sim\sim p))}$ ~~(4)~~

~~By Axiom 3~~ $\cancel{\sim\sim p \to (\sim\sim p \to p)}$

By Axiom 3 $(\sim p \to \sim(\sim\sim p)) \to (\sim\sim p \to p)$ --- (4)

By Transitivity 3, 4; $\sim\sim p \to (\sim\sim p \to p)$ --- (5)

By Axiom 2 $(\sim\sim p \to (\sim\sim p \to p)) \to ((\sim\sim p \to \sim\sim p) \to (\sim\sim p \to p))$ --- (6)

By 5, 6, Modus Ponens. $(\sim\sim p \to \sim\sim p) \to (\sim\sim p \to p)$ --- (7)

$(\sim\sim p \to \sim\sim p)$ --- (8)

By Axiom 1 $\sim\sim p \to ((\varphi \to \sim\sim p) \to \sim\sim p)$

By Axiom 2. $(\sim\sim p \to ((\varphi \to \sim\sim p) \to \sim\sim p)) \to ((\sim\sim p \to (\varphi \to \sim\sim p)) \to (\sim\sim p \to \sim\sim p))$ --- (9)

By 8, 9, Modus Ponens. $(\sim\sim p \to (\varphi \to \sim\sim p)) \to (\sim\sim p \to \sim\sim p)$ --- (10)

By Axiom 1 $\sim\sim p \to (\varphi \to \sim\sim p)$ --- (11)

By 10, 11, M.P $\Rightarrow$ $\sim\sim p \to \sim\sim p$ (12)

By 7, 12, M.P $\Rightarrow$ we have $(\sim\sim p \to p)$ proved