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MARKS OBTAINED

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The three basic logical operators $\{\sim, \wedge, \vee\}$ called natural operators in propositional calculus.

Def^n:- A set of logical operators is adequete if operats in that set is able to express any other operators in terms of themselves.

For example:- The operator sets $\{\wedge, \sim\}$ and $\{\vee, \sim\}$ can represent any other logical operators, and therefore adequet.

Def^n:- A formula is said to be ~~said~~ valid if and only if it is true under all interpretations. That is a formula is valid if it is a tautology.

Consistency and Inconsistency:

Def^n:- An interpretation is called a model of a formula if it is evaluated to be true under that interpretation.

Def^n:- If there exists an interpretation ~~which~~ for ~~the formula~~ ~~true~~ for which the formula is true then the formula is said to be consistent.

Def^n:- A formula is said to inconsistent if and only if the formula is false under all interpretations. Then the formula is also called a contradiction.

Def^n:-

A set of formulae is said to be mutually consistent if and only if they are all true simultaneously. for some interpretation(same set of interpretations).

Def^n:-

A set of formulae is said to be mutually inconsistent if and only if the conjunction (and) of formulae is always false.

Eg:- 1. Show that the formula $p \vee q \rightarrow \sim p$ is consistent with truth table.

Find its model.

Ans. By drawing its truth table we have:

p	q	$p \vee q$	$\sim p$	$p \vee q \rightarrow \sim p$
T	T	T	F	F
T	F	T	F	F
F	T	T	T	T
F	F	F	T	T

From these table we can say that the formula $p \vee q \rightarrow \sim p$ is consistent and Two model of its are $\{p=F, q=T\}$, $\{p=F, q=F\}$.

Eg:- 2. Show that the formula $p \wedge \sim p$ is inconsistent with truth table.

Ans

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

So from this table we can say that the formula $p \wedge \sim p$ is false under all

interpretation of the formula $P \wedge \sim P$.
therefore the formula $P \wedge \sim P$ is false.

Eg:- Show that the following set of formula ~~are~~ is mutually consistent.

$$S = \{ P \wedge Q,\ P \vee Q,\ P \rightarrow Q \}$$

Ans Let the truth table for S is as below.

P	Q	$P \wedge Q$	$P \vee Q$	$P \rightarrow Q$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

From this we conclude that for $\{P = T, Q = T\}$ assignment ~~S~~ formulae in S set are mutually consistent.

Eg:- Show that following set of formulae are mutually inconsistent.

$$S = \{ P \rightarrow (\sim R \rightarrow Q),\ P \rightarrow \sim R,\ \sim (P \rightarrow Q) \}$$

∴ let $\alpha = P \rightarrow (\sim R \rightarrow Q) \wedge (P \rightarrow \sim R) \wedge (\sim (P \rightarrow Q))$

Ans By forming truth table of S we have

α	P	Q	R	$\sim R$	$P \rightarrow Q$	$P \rightarrow \sim R$	$\sim (P \rightarrow Q)$	$\sim R \rightarrow Q$	$P \rightarrow (\sim R \rightarrow Q)$
F	T	T	T	F	T	F	F	T	T
F	T	T	F	T	T	T	F	T	T
F	T	F	T	F	F	F	T	T	T
F	T	F	F	T	F	T	T	F	F
F	F	T	T	F	T	T	F	T	T
F	F	T	F	T	T	T	F	T	T
F	F	F	T	F	T	T	F	T	T
F	F	F	F	T	T	T	F	F	T

So under all interpretation conjunction of set of formula is false.
∴ the set of formula 'S' is mutually inconsistent.

Def^n: If Σ be a set of formulae then another formula σ is said to logical consequence of Σ, if any of the interpretation I, which satisfies all formula belongs to Σ, also satisfies σ. And it is denoted as $\Sigma \models \sigma$

Eg:- let $\Sigma = \{P \vee Q \to R \wedge Q,\ P \vee Q\}$, then show that $R \wedge Q$ is logical consequence of Σ.

Ans. By forming truth table we have,

	P	Q	R	$P \vee Q$	$R \wedge Q$	$P \vee Q \to R \wedge Q$
*	T	T	T	T	T	T
	T	T	F	T	F	F
	T	F	T	T	F	F
	T	F	F	T	F	F
•	F	T	T	T	T	T
	F	T	F	T	F	F
	F	F	T	F	F	T
	F	F	F	F	F	T

From table we can see that the interpretation $\{P = T, Q = T, R = T\}$ and $\{P = F, Q = T, R = T\}$ satisfies Σ as well as $R \wedge Q$, therefore we can say that $R \wedge Q$ is the logical consequence of Σ.

Transforming english sentences to formal

Q:

Ans. Let the assumptions as follows:

R : It rains;
H : I stay at home;

W: I will wet
C : Trip is cancelled

Then the sentences

1) If it rains and I stay at home then I won't be wet.

2) I will be wet if it rains.

3) If it rains then the trip is cancelled

4) Either it does not rain or I am staying at home

5) Whether or not the trip is cancelled, I am staying home if it rains.

can be converted into following formulae

From 1) $R \land H \longrightarrow \sim W$

2) $R \longrightarrow W$

3) $R \rightarrow C$

4) $\sim R \lor H$

5) $(C \lor \sim C) \land R \longrightarrow H \cong R \rightarrow H$

6. If it rains and the trip is not cancelled or I don't stay at home then I will be wet.

$(R \land \sim C) \lor \sim H \longrightarrow W$

Ans $(R \land \sim C) \lor \sim H \longrightarrow W$

$\cancel{X}$ or $R \land (\sim C \lor \sim H) \longrightarrow W \rightarrow$ it is more desirable.

7. "If it is Sunday and nice weather then we will go for swimming: Today is sunday. Weather is nice"

Then show that "we will go for swimming", is the logical consequence of the above text.

Ans. Let the given sentences are represented by propositions as follows.

$P :\Rightarrow$ It is sunday

$Q \Rightarrow$ weather is nice

$R :\Rightarrow$ we will go for swimming.

Then first sentence can be written as

$P \wedge Q \rightarrow R$

The 2nd and third one will be simple written as

$P = T$ [Today is Sunday]

$Q = T$ [weather is nice]

Now, By truth table

P	Q	R	$P \wedge Q$	$P \wedge Q \rightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

Now from table we can say that

: The set of formula $P, Q, P \wedge Q \rightarrow R$ simultaneously satisfies with $P = T, Q = T, R = T$

So if $P, Q, P \wedge Q \rightarrow R$ all are satisfies with $P = T, Q = T, R = T$,

and R is is also satisfies with $P = T, Q = T, R = T$

Condition then we can say that R is the logical consequence of $P \wedge Q \rightarrow R$.