① **DURGAPUR INSTITUTE OF ADVANCED TECHNOLOGY & MANAGEMENT**

Internal Test of 20

BRANCH - SEMESTER -

NAME

NAME OF PAPER PAPER CODE

COLLEGE ROLL No. DATE

MARKS OBTAINED Signature of the Invigilator

Signature of Examiner

Propositional logic

A proposition is a statement which is either true or false but not both simultaneously.

An Inference is is a process by which one proposition is arrived at and affirmed on the basis of one or more other propositions accepted at the starting point.

Predicate logic is an extension of propositional logic in which we can define relation with constants, variables and functions, as its arguments.

For example :→ Sun rises in the east

↳ is a proposition.

but x is greater than 3

↳ is a predicate.

Propositional calculus is a thing which consists of the following.

1. A set of propositional symbols P, Q, R called atoms.
2. logical constants True (T), false (F).
3. A set of logical operators.

∧, ∨, ~, →, ⟷ → double implecation

conjunction, disjunction, Negation, implecation

Well formed formula (wff's).

A well formed formula in Propositional Calculus is defined as follows, recursively:

1. An atom is a well formed formula
2. If α is a well formed formula then $\sim\alpha$, is also a well formed formula
3. If α and β are two well formed formulae then $(\alpha \land \beta)$, $(\alpha \lor \beta)$, $(\alpha \rightarrow \beta)$, $(\alpha \leftrightarrow \beta)$ are also well formed formula.
4. A propositional expression is a well formed formula if and only if it can be obtained by using above rules.

For example

$\sim(\sim p \lor \sim q)$ $\longrightarrow$ is a well formed formula

but $(\sim p \lor (\land q)) \land (q \lor p \sim)$ is not a wff.

Truth table

A truth table is a table by which we can obtain ~~all possible~~ truth values of a well formed formula for its every instance or interpretation.

The logical operations of basic wff's are as follows

p	q	$\sim p$	$p \land q$	$p \lor q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

Operators precedence are as follows.

$\sim, \land, \lor, \rightarrow, \leftrightarrow$ $\longrightarrow$ a left to right according to priority (precedence).

Eg:- Compute truth table for the formula

$p \vee q \rightarrow (\sim q \rightarrow \sim p)$

Ans

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim q \rightarrow \sim p$	$p \vee q \rightarrow (\sim q \rightarrow \sim p)$
			F	T	T	T
T	T	F			F	F
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	F	T	T

Eg Evaluate the truth value of the formula $(P \rightarrow q) \wedge R \leftrightarrow (P \wedge q \vee S)$, if atoms in a set $\{P, q, R, S\}$ as $\{F, T, F, T\}$

Ans The formula :-

$$(P \rightarrow q) \wedge R \leftrightarrow (P \wedge q \vee S)$$

$$= (F \rightarrow T) \wedge F \leftrightarrow (F \wedge T \vee T)$$

$$= (T \wedge F) \leftrightarrow (F \vee T)$$

$$= F \leftrightarrow T$$

$$= F. \quad \text{Ans}$$

Def^n A truth assignment (or valuation) is a function that assigns to each atom P a unique truth value either true (T) or false (F).

Def^n. Each row of a truth table for a given formula is called its interpretation.

Def^n A formula α is called tautology if and only if α is true for all interpretations.

For example, $P \vee \sim P$ is a ~~tautolog~~ tautology.

Defⁿ: If α and β be two formulae. Then they are said to be <u>logically equivalent</u> if and only if the truth values of α and β are same under all interpretations. It is denoted by $\alpha \equiv \beta$

<u>For example:</u>

Show that $\sim p \vee q$ and $p \to q$ are logically equivalent using truth table.

<u>Ans.</u>

p	q	$\sim p$	$\sim p \vee q$	$p \to q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Therefore from above truth table we see that $\sim p \vee q$ and $p \to q$ formulae are same under all interpretation, therefore we can write

$$\sim p \vee q \cong p \to q \quad \text{logically equivalent.}$$

<u>Equivalence laws</u>

<u>Commutation</u>

1. $p \wedge q \cong q \wedge p$
2. $p \vee q \cong q \vee p$.

<u>Association</u>

1. $p \wedge (q \wedge R) \cong (p \wedge q) \wedge R$.
2. $p \vee (q \vee R) \cong (p \vee q) \vee R$.

<u>Double Negation</u>

$\sim(\sim p) \cong p$

<u>Distribution</u>

1. $p \wedge (q \vee R) \cong (p \wedge q) \vee (p \wedge R)$
2. $p \vee (q \wedge R) \cong (p \vee q) \wedge (p \vee R)$

<u>De morgan's laws.</u>

1. $\sim(p \wedge q) \cong \sim p \vee \sim q$
2. $\sim(p \vee q) \cong \sim p \wedge \sim q$.

<u>law of excluded middle</u>

$p \vee \sim p \cong T$ (True).

<u>law of contradiction</u>

$p \wedge \sim p \cong F$ (false)

Idempotence law	**Absorption law**
1. $P \wedge P \cong P$	1. $P \wedge (P \vee q) \cong P$
2. $P \vee P \cong P$	2. $P \vee (P \wedge q) \cong P$
	3. $P \vee (\sim P \wedge q) \cong P \vee q$
	4. $P \wedge (\sim P \vee q) \cong P \wedge q$.

Commonly used equivalence relations

$P \wedge F \cong F$	$P \vee T \cong T$
$P \wedge T \cong P$	$P \to q \cong \sim p \vee q$
$P \vee F \cong P$	$P \leftrightarrow q \cong (P \to q) \wedge (q \to P)$
	$\cong (P \wedge q) \vee (\sim P \wedge \sim q)$.

Eg. Show that following formulae are logically equivalent.

1. $(P \wedge q) \vee (P \wedge \sim q) \cong P$
2. $\sim P \to \sim (P \to \sim q) \cong P$.

Ans

1. $(P \wedge q) \vee (P \wedge \sim q)$ [The L.H.S]

$\cong P \wedge (q \vee \sim q)$ [By distributive law]

$\cong P \wedge T$ [By law of excluded middle]

$\cong P.$ [By basic def^n of '$\wedge$']

$\therefore (P \wedge q) \vee (P \wedge \sim q) \cong P$ proved.

---

2. $\sim P \to \sim (P \to \sim q)$ [The L.H.S]

$\cong \sim p \to \sim (\sim P \vee \sim q)$ [As $P \to q \cong \sim p \vee q$]

~~$\cong \sim p \to \sim(\sim P) \wedge \sim(\sim q)$~~

$\cong \sim p \to \sim(\sim (P \wedge q))$ [By de morgan's law]

$\cong \sim p \to (P \wedge q)$ [By double negation law]

$\cong \sim(\sim P) \vee (P \wedge q)$ [As $P \to q \cong \sim p \vee q$]

$\cong P \vee (P \wedge q)$ [By double negation law]

$\cong P$ [By absorption law]

$\therefore \sim P \to \sim (P \to \sim q) \cong P$ proved.

There are two extra rules which are used to simplify the formula.

1. Insertion rule:

If 'α' and β' are two tautologies and 'P' be an atom occurring in 'α', and is replaced by the formula 'β' in all occurance of P in 'α', then the newly created formula is also a tautology.

For example,

Let $\alpha: P \vee (\sim P \wedge Q) \leftrightarrow (P \vee Q)$.

and $\beta: R \vee \sim R$

then 'P' in α, if replaced by β' we get,

$\delta: (R \vee \sim R) \vee (\sim (R \vee \sim R) \wedge Q) \leftrightarrow ((R \vee \sim R) \vee Q)$.

Then according to insertion rule δ is a tautology.

2. Substitution Rule

If α is a formula and β' is another formula occurring in 'α', if 'λ' is another formula such that $\beta \cong \lambda$, then new formula δ, created by replacing at least one occurance of 'β' in 'α' by 'λ' is logically equivalent to 'α'. i.e $\alpha \cong \delta$.

For example,

Let $\alpha: P \to (R \to Q) \leftrightarrow \sim P \vee (R \to Q)$, consider sub formula $\beta: R \to Q$. and other formula $\lambda: \sim R \vee Q$

therefore $\beta \cong \lambda$.

Now new formula created from α by replacing β by λ, we get,

$\delta_1: P \to (\sim R \vee Q) \leftrightarrow \sim P \vee (\sim R \vee Q)$

$\delta_2: P \to (\sim R \vee Q) \leftrightarrow \sim P \vee (R \to Q)$

$\delta_3: P \to (R \to Q) \leftrightarrow \sim P \vee (\sim R \vee Q)$

Therefore $\delta_1, \delta_2, \delta_3$ all are equivalent to α.